UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

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UNIVERSITY *of* CAMBRIDGE
International Examinations

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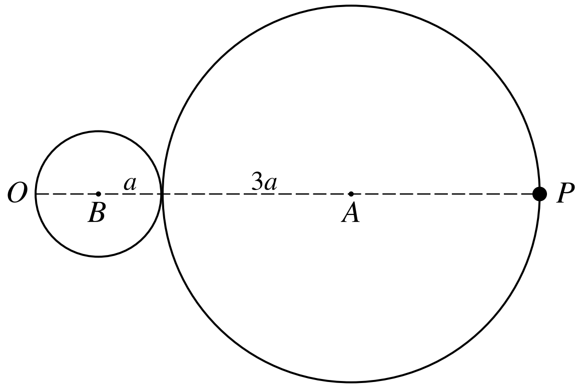
1 Three small spheres, A, B and C, of masses m, km and $6m$ respectively, have the same radius. They are at rest on a smooth horizontal surface, in a straight line with B between A and C. The coefficient of restitution between A and B is $\frac{1}{2}$ and the coefficient of restitution between B and C is e. Sphere A is projected towards B with speed u and is brought to rest by the subsequent collision. Show that $k = 2$. [3]

Given that there are no further collisions after B has collided with C, show that $e \leqslant \frac{1}{3}$. [5]

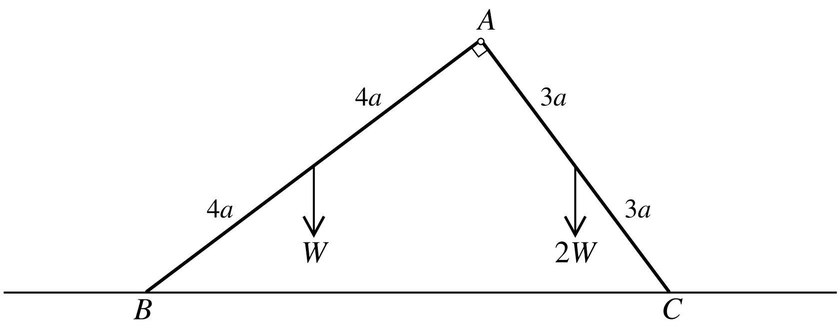
2



A uniform circular disc with centre A has mass M and radius $3a$. A second uniform circular disc with centre B has mass $\frac{1}{9}M$ and radius a. The two discs are rigidly joined together so that they lie in the same plane with their circumferences touching. The line of centres meets the circumference of the larger disc at P and the circumference of the smaller disc at O. A particle of mass $\frac{1}{3}M$ is attached at P (see diagram). Show that the moment of inertia of the system about an axis through O, perpendicular to the plane of the discs, is $51Ma^2$. [6]

The system is free to rotate about a fixed horizontal axis through O, perpendicular to the plane of the discs. The system is held with OP horizontal and is then released from rest. Given that $a = 0.5$ m, find the greatest speed of P in the subsequent motion, giving your answer correct to 2 significant figures. [5]

3



The diagram shows two uniform rods BA and AC, smoothly hinged at A. The rod BA has length $8a$ and weight W; the rod AC has length $6a$ and weight $2W$. The rods are in equilibrium in a vertical plane with B and C resting on a rough horizontal floor and angle CAB equal to $90°$. Show that the normal contact force at B is $\frac{26}{25}W$. [4]

The coefficient of friction between each rod and the floor is μ. Find the least possible value of μ. [8]

4 A particle P of mass m is suspended from a fixed point O by a light inextensible string of length a. When hanging at rest under gravity, P is given a horizontal velocity of magnitude $\sqrt{(3ag)}$ and subsequently moves freely in a vertical circle. Show that the tension T in the string when OP makes an angle θ with the downward vertical is given by

$$T = mg(1 + 3\cos\theta).$$ [4]

When the string is horizontal, it comes into contact with a small smooth peg Q which is at the same horizontal level as O and at a distance x from O, where $x < a$. Given that P completes a vertical circle about Q, find the least possible value of x. [8]

5 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} 0.01\mathrm{e}^{-0.01x} & x \geqslant 0, \\ 0 & x < 0. \end{cases}$$

(i) State the value of $\mathrm{E}(X)$. [1]

(ii) Find the median value of X. [3]

(iii) Find the probability that X lies between the median and the mean. [2]

6 The independent random variables X and Y have distributions with the same variance σ^2. Random samples of 5 observations of X and n observations of Y are made and the results are summarised by

$$\Sigma x = 5.5, \quad \Sigma x^2 = 15.05, \quad \Sigma y = 8.0, \quad \Sigma y^2 = 36.4.$$

Given that the pooled estimate of σ^2 is 3, find the value of n. [7]

7 A fair die is thrown until a 6 appears for the first time. Assuming that the throws are independent, find

(i) the probability that exactly 5 throws are needed, [3]

(ii) the probability that fewer than 8 throws are needed, [2]

(iii) the least integer n such that the probability of obtaining a 6 before the nth throw is at least 0.99. [3]

8 A company decides that its employees should follow an exercise programme for 30 minutes each day, with the aim that they lose weight and increase productivity. The weights, in kg, of a random sample of 8 employees at the start of the programme and after following the programme for 6 weeks are shown in the table.

Employee	A	B	C	D	E	F	G	H
Weight before (kg)	98.6	87.3	90.4	85.2	100.5	92.4	89.9	91.3
Weight after (kg)	93.5	85.2	88.2	84.6	95.4	89.3	86.0	87.6

Assuming that loss in weight is normally distributed, find a 95% confidence interval for the mean loss in weight of the company's employees. [6]

Test at the 5% significance level whether, after the exercise programme, there is a reduction of more than 2.5 kg in the population mean weight. [5]

9 The marks achieved by a random sample of 15 college students in a Physics examination (x) and in a General Studies examination (y) are summarised as follows.

$$\Sigma x = 752 \qquad \Sigma x^2 = 38\,814 \qquad \Sigma y = 773 \qquad \Sigma y^2 = 45\,351 \qquad \Sigma xy = 40\,236$$

(i) Find the mean values, $\bar{x}$ and $\bar{y}$. [1]

(ii) Another college student achieved a mark of 56 in the General Studies examination, but was unable to take the Physics examination. Use the equation of a suitable regression line to estimate the mark that the student would have obtained in the Physics examination. [4]

(iii) Find the product moment correlation coefficient for the given data. [2]

(iv) Stating your hypotheses, test at the 5% level of significance whether there is a non-zero product moment correlation coefficient between examination marks in Physics and in General Studies achieved by college students. [4]

10 Answer only **one** of the following two alternatives.

EITHER

One end of a light elastic string is attached to a fixed point O. A particle of mass m is attached to the other end of the string and hangs freely under gravity. In the equilibrium position, the extension of the string is d. Show that the period of small vertical oscillations about the equilibrium position is $2\pi\sqrt{\left(\dfrac{d}{g}\right)}$. [5]

The particle is now pulled down and released from rest at a distance $2d$ below the equilibrium position. Given that the particle does not reach O in the subsequent motion, show that the time taken until the particle first comes to instantaneous rest is $\left(\sqrt{3} + \frac{2}{3}\pi\right)\sqrt{\left(\dfrac{d}{g}\right)}$. [9]

OR

A family was asked to record the number of letters delivered to their house on each of 200 randomly chosen weekdays. The results are summarised in the following table.

Number of letters	0	1	2	3	4	5	$\geqslant 6$
Number of days	57	60	53	25	4	1	0

It is suggested that the number of letters delivered each weekday has a Poisson distribution. By finding the mean and variance for this sample, comment on the appropriateness of this suggestion. [3]

The following table includes some of the expected values, correct to 3 decimal places, using a Poisson distribution with mean equal to the sample mean for the above data.

Number of letters	0	1	2	3	4	5	$\geqslant 6$
Expected number of days	53.964	70.693	p	q	6.622	1.735	0.463

(i) Show that $p = 46.304$, correct to 3 decimal places, and find q. [2]

(ii) Carry out a goodness of fit test at the 10% significance level. [9]

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